AP Statistics
Exam: Final Exam
Instructor's Version

Free-Response Questions:

Note: The first five of these questions are worth 4 points each. The sixth question is worth 8 points. Use your discretion in awarding points. The free-response portion of the exam is worth 112 points, so multiply the total points by 4 to calculate the final grade.

1. You have two six-sided dice. Die *A* has 2 twos, 1 three, 1 five, 1 ten, and 1 fourteen on its faces. Die *B* has a one, a three, a five, a seven, a nine, and an eleven on its faces.

A. What are the average expected values for die *A* and die *B*?

Answer:
Die *A*: 2(1/3) + 3(1/6) + 5(1/6) + 10(1/6) + 14(1/6) = 6
Die *B*: (1 + 3 + 5 + 7 + 9 + 11)(1/6) = 6

B. On any given roll of both dice, what is the probability that the number showing on die *A* will be greater than the number on die *B*? What is the probability that the number showing on die *B* will be greater?

Answer:
The following table shows the outcomes for all possible combinations of faces on die *A* and die *B*.

DIE *A*

DIE *B*	**2**	**2**	**3**	**5**	**10**	**14**
1	*A*	*A*	*A*	*A*	*A*	*A*
3	*B*	*B*	TIE	*A*	*A*	*A*
5	*B*	*B*	*B*	TIE	*A*	*A*
7	*B*	*B*	*B*	*B*	*A*	*A*
9	*B*	*B*	*B*	*B*	*A*	*A*
11	*B*	*B*	*B*	*B*	*B*	*A*

We see that, of the 34 possible outcomes that are not ties, die *A* has the higher number 16 times, and die *B* has the higher number 18 times. Thus:

P(die *A* has larger number) = 16/34 = 8/17

P(die *B* has larger number) = 18/34 = 9/17

C. Considering your answers for parts A and B, which die would you choose to roll, and why, if your goal was (i) to achieve a higher score than an opponent rolling the other die, and (ii) to achieve the highest possible score on a single roll of the die?

Answer:
(i) We need to look at head-to-head competition between the two dice. On average, die *A* will win 16 times and die *B* will win 18 times in 34 rolls, so we would prefer to play with die *B*.
(ii) Since we are only interested in the high score on a single roll, we need the die with the highest expected value. Since die *A* and die *B* have an equal expected value, though, we would have no preference in this case.

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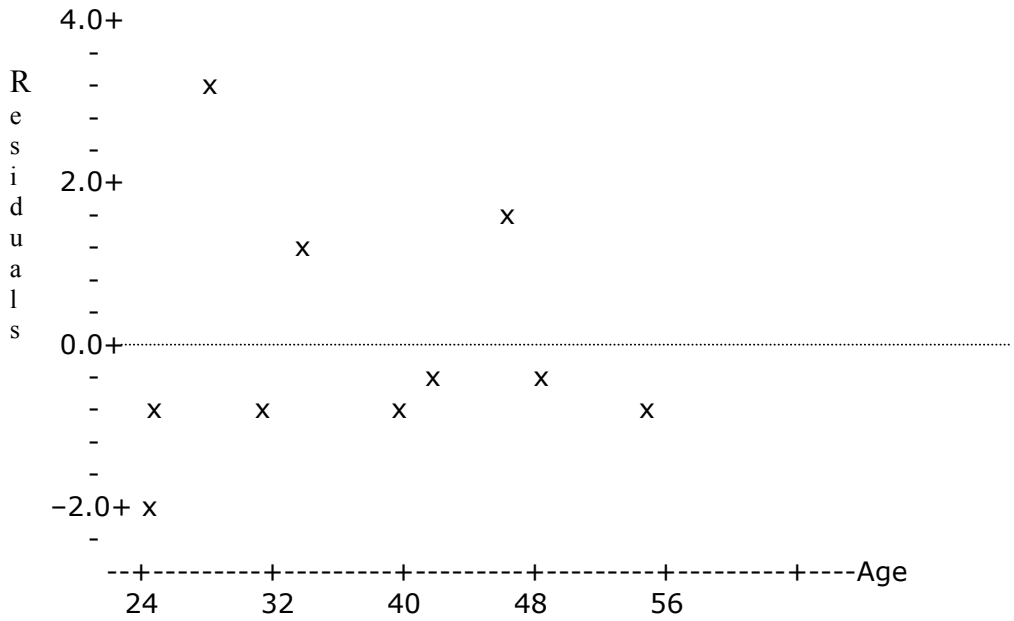
2. You conduct a study to determine whether there is a relationship between how old a person is when they divorce and the amount of time before he or she remarries. Some computer output from the analysis and a residual plot of the data are presented below. (Values are in years.)

Regression Analysis

The regression equation is:
Time = 0.03 + 0.124 Age

Predictor	Coef	Stdev	t-ratio	p
Constant	0.034	1.962	0.02	0.987
Age	0.12378	0.04963	2.49	0.037

s = 1.659 R-sq = 43.7% R-sq(adj) = 36.7%



A. What is the predicted number of years a 40-year-old will wait before remarrying?

Answer:
Time = .03 + .124(40) = 4.99

B. Given that the study found that the average 40-year-old waits 4 years before remarrying, what is the value of the residual for a 40-year-old?

Answer:
Residual = actual – predicted = 4.0 – 4.99 = –.99

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C. Based on the printout and the residual plot, does it appear that "Time" and "Age" have a linear relationship? If so, how strong does the predictive relationship appear to be?

Answer:
The points in the residual plot are scattered more or less randomly about the $y = 0$ line. The residuals above the line are greater than those below the line, but there does not appear to be a distinct pattern in the points. Thus, a linear model is appropriate. Also, the *P*-value for the slope of the regression line is .037, which is good evidence that a linear relationship exists.

Though it is reasonable to use a linear model here, the predictive strength of the model is not great. That $r^2 = .437$ implies that less than half of the variation in the time to remarriage can be explained by the linear relationship between age at time of divorce and the amount of time before remarrying. Three of the residuals are much larger than the rest, which explains some of the weakness of the correlation.

In conclusion, we can say this relationship is linear, but we should be careful in using it to make predictions.

3. You believe that requiring students to attend a series of presentations on study skills will improve their grades. You are concerned that boys and girls might react differently to such a series, and that students in grades 9 and 10 might react differently than those in grades 11 and 12. You set up an experiment involving 100 students in grades 9 and 10 and 100 students in grades 11 and 12. (Assume there are equal numbers of girls and boys in each grade.) You intend to measure improvement based on pre- and post-treatment grade-point averages.

A. Describe the design of an experiment to help you determine if a set of presentations on study skills is effective in improving grades.

Answer:
You would first block by grade level, one block being composed of grades 9 and 10 and one block being composed of grades 11 and 12. Within each of these, you would block again, this time by gender. This would give you the following four blocks:

- Boys in grades 9 and 10
- Girls in grades 9 and 10
- Boys in grades 11 and 12
- Girls in grades 11 and 12

Assuming that each of these blocks contains approximately 50 students, you would randomly assign 25 students to the treatment group (the group to attend the series of presentations) and 25 students to the control group (the group that will not attend the presentations). Once the study is completed, you would compare the pre- and post-treatment grade-point averages of the two groups within each block to see if there is a difference.

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B. Suppose you have 50 students and need to assign them to two groups of equal size. Describe a randomization procedure that would achieve this.

Answer:
Assign each student a number from 1 to 50. Use a random number generator to pick 25 numbers from 1 to 50. The students whose assigned numbers matched the randomly generated numbers would be the treatment group, and the remaining 25 would be the control group.

C. Suppose you have 50 students and need to assign them to two independent groups. Describe a randomization procedure that would achieve this.

Answer:
Pick a student and flip a coin. If the coin comes up heads, the student is in the treatment group. If it comes up tails, the student is in the control group. Note that it is unlikely that the two groups would be of equal size under the independence requirement.

D. Describe an appropriate method of analysis for the data you collect from this experiment.

Answer:
Since we are using pre- and post-treatment grade-point averages on the same group of students, the data are paired. Thus, a matched-pairs *t* test is appropriate. We would find the differences between the scores by subtracting the pre-treatment scores from the post-treatment scores and testing the hypothesis $\mu_d = 0$. If you believed, before you collected the data, that post-treatment scores would be higher, the alternative hypothesis would be $H_a : \mu_d > 0$. In this case, you would use a one-sample *t* test to produce a *P*-value upon which you would base your conclusion.

4. In a large public university, the number of sections of calculus to be offered in a given year depends on the proportion of students completing the prerequisite course. The mathematics department must decide how many calculus sections to create **before** students begin to register for classes. To help them decide, the department decides to survey a random sample of students currently enrolled in the prerequisite course. If more than 80% of these students plan to take calculus, the department will need to increase the number of sections offered. They survey 130 students, 125 of whom respond. Of these 125, 108 say they intend to take calculus in the following year.

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A. On the basis of these data, would you recommend to the mathematics department that they increase the number of sections of calculus for the following year? Use appropriate statistical evidence to support your answer.

Answer:
H_0 : P = .8 (The proportion of students planning to take calculus is .8.)
H_a : P > .8 (The proportion of students planning to take calculus is greater than .8.)

Since (.8)(125) and (.2)(125) are both greater than 10, we are justified in using a one-proportion *z*-test.

$$\hat{p} = \frac{108}{125} = .864;\ z = \frac{.864 - .8}{\sqrt{\frac{.8(1-.8)}{125}}} = \frac{.064}{.036} = 1.78 \rightarrow P = .0375$$

(We would get .0368 if we used the TI-83.)

This *P*-value is low enough that we have strong evidence against the null hypothesis that the proportion of students planning to take calculus is .8. It seems likely that the true proportion is higher, and that additional sections will be needed. (If a student used a significance level of .01, they would have concluded that there were not grounds to reject the null, and so additional sections would not be needed.)

B. Suppose the five missing responses from the original 130 had simply gotten lost on the department secretary's desk while the rest were being analyzed. In what way could the inclusion of these five responses have affected your recommendation?

Answer:
If the recommendation was to add sections, and all five of the new responses had said they intended to take calculus, then the recommendation would not have changed.

(This would change the *P*-value to .024, based on $\hat{p} = \frac{113}{130} = .869$.
The recommendation of the students who used a .01 significance level would not have changed either, even though in this case it is even less likely that the true proportion is .8.)

However, in the worst case, all five might have said they did not intend to take calculus. That would give us

$$\hat{p} = \frac{108}{130} = .831,\ z = \frac{.831 - .8}{\sqrt{\frac{.8(1-.8)}{130}}} = \frac{.031}{.035} = .89 \rightarrow P = .19.$$

This finding is not unlikely if the true proportion is .8, and the likely recommendation would have been to not create additional sections.

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5. Theorists believe that early exposure to structured exercise will increase artistic ability. A group of 15 pairs of identical twins are selected for an experiment to test this theory. One of each pair is randomly assigned to a twice-weekly exercise program for one year. The other twin exercises as much or as little as he or she wishes. After one year, each twin is given the same test of artistic ability (higher scores indicating higher ability). The persons judging the test do not know which half of each pair has participated in the exercise program. The results are as follows:

Twin Set	Group *A* (Exercise Program)	Group *B* (No Exercise Program)
1	86	85
2	49	53
3	77	70
4	63	60
5	59	53
6	99	84
7	77	79
8	83	85
9	67	65
10	73	70
11	78	72
12	74	70
13	86	88
14	73	71
15	69	60

The mean score for those having participated in the exercise program is 74.2 with a standard deviation of 12.17. The mean score for those that did not participate in the exercise program is 71 with a standard deviation of 11.48. The difference for the scores of Group *A* minus Group *B* has a mean of 3.2 with a standard deviation of 4.96.

Does it appear that early exposure to exercise increases artistic ability? Use appropriate statistical evidence to support your answer.

Answer:
Note: It is important that the student recognizes that this is a matched pairs problem. A maximum of half-credit is possible if they treat this as an independent two-sample test.

$H_0 : \mu_d = 0$ (There is no difference between the mean score for those who participated in the exercise program and the mean score for those who did not.)
$H_a : \mu_d > 0$ (The mean score for those who participated in the exercise program will be greater than the mean score for those who did not.)

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We can assume that this set of scores represents a simple random sample from a normal population, since a boxplot (or scatterplot, dotplot, normal quantile plot – the student should *show* this, not just *claim* it) of the differences shows no significant departures from normal. Thus, a one-sample *t* test is appropriate.

$t = \dfrac{3.2 - 0}{4.96/\sqrt{15}} = 2.50$, $df = 14$ ➔ $.01 < p < .02$

($p = .013$)

Note: Students should not construct a confidence interval here, since the alternative hypothesis is one-sided.

A *P*-value this low provides strong evidence against the null hypothesis. Thus, we have good evidence that early exposure to exercise is effective in increasing artistic ability as measured by this experiment.

Note: It is not necessary to state a significance level for this problem. If the student did state a significance level of .05, the conclusion would be as shown here. If the student stated a significance level of .01, they should not reject the null.

6. To be legal for certain types of prize competitions, bowling balls must be very close to 16 lb. The weights of bowling balls from a certain manufacturer are known to be normally distributed with a mean of 16 lb. and a standard deviation of .25 lb. A ball will be rejected if it weighs less than 15.68 lb.

A. What is the probability that a ball will be rejected? Round your answer to two significant digits.

Answer:
The distribution of weights of bowling balls has $N(16, .25)$

$z_{15.68} = \dfrac{15.68 - 16}{.25} = -1.28$ ➔ $P = .10$

B. Describe how you would use a table of random digits (comprised of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) to simulate how long it would take to identify three defective bowling balls.

Answer:
Let "1" stand for finding a defective ball, and let every other value stand for one that is not defective. Randomly choose a starting point in the table of random digits and proceed through the table until you get three 1s. The statistic of interest is the number of digits you got before you got the three 1s.

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C. Use the table below to perform the simulation you described in part B, this time for three defective bowling balls. Repeat your simulation three times, starting with the first digit of the first line and proceeding left to right, starting again at the extreme left of each successive line. What is the average waiting-time based on your three simulations?

77014 21414 95729 01392 37814 22931 94998 56569 30213 03469
16334 43057 03297 61609 68462 26199 98324 41436 96050 95744
98563 56006 93060 29402 76577 39814 75704 26127 42577 17458
25883 51840 45515 18925 46458 61380 79369 01710 93720 73046
16434 57044 23969 78022 67976 23279 67173 44918 91684 94775

Answer:
The 1s are boldfaced and underlined. The divisions between blocks where three 1s are found are separated by slashes.

770**1**4 2**1**4**1**//4 95729 0**1**392 378**1**4 2293**1**//94998 56569 302**1**3 03469
16334 43057 03297 6**1**//609 68462 26199 98324 41436 96050 95744
98563 56006 93060 29402 76577 39814 75704 26127 42577 17458
25883 51840 45515 18925 46458 61380 79369 01710 93720 73046
16434 57044 23969 78022 67976 23279 67173 44918 91684 94775

The waiting times are 9, 21, and 37. The average number of bowling balls considered before three defectives are found is 22.3.

D. What is the probability that three or more defective bowing balls are found in the first 25 examined? Do *not* do the calculation, but do write out the mathematical expression you would use to answer the question.

Answer:
Assuming that the probability of each bowling ball being defective is independent of any other ball being defective, the situation is binomial. Let X be the count of successes (that is, defective bowling balls) in 25 observations of a binomial event that occurs with probability .1. Since 25(.1) is not greater than 10, the student should not attempt to do a normal approximation to this binomial. The binomial expression to be calculated should look like this:

$$P(X \geq 3) = 1 - \left(\binom{25}{0}(.1)^0(.9)^{25} + \binom{25}{1}(.1)^1(.9)^{24} + \binom{25}{2}(.1)^2(.9)^{23} \right).$$

(The value of this expression is approximately .46.)

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